Chapter 2 Kinematics of Linear Motion

Curriculum Specification	**Remarks**		
	Before	**After**	**Revision**
2.1 Linear motion			
a) Define i. instantaneous velocity, average velocity and uniform velocity ii. instantaneous acceleration, average acceleration and uniform acceleration (C1, C2)			
b) Discuss the physical meaning of displacement-time, velocity-time and acceleration-time graphs. (C1, C2)			
c) Determine the distance travelled displacement, velocity and acceleration from appropriate graphs. (C3, C4)			
2.2 Uniform accelerated motion			
a) Apply equations of motion with uniform acceleration: $\vec{v} = \vec{u} + \vec{a}t$ $\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$ $\vec{v}^2 = \vec{u}^2 + 2\vec{a} \bullet \vec{s}$ (C3, C4)			
2.3 Projectile motions			
a) Describe projectile motion launched at an angle, θ as well as special cases when $\theta = 0°$ and $\theta = 90°$ (free fall). (C1, C2)			
b) Solve problems related to projectile motion. (C3, C4)			
c) Determine the acceleration due to gravity, g using free fall and projectile motion. (Experiment 2: Free fall and projectile motion) (C1, C2, C3, C4)			

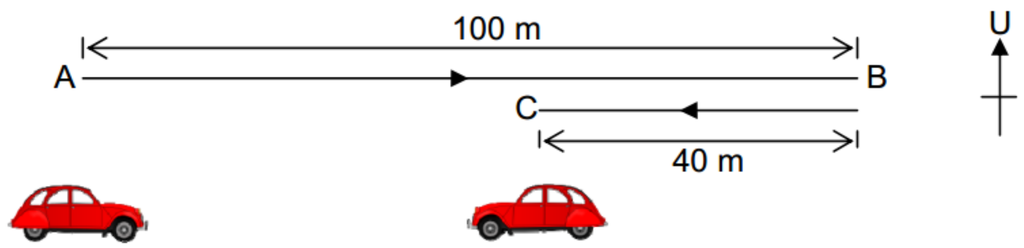
Revision: Distance and Displacement

There are two aspects to any motion:

- **Kinematics** deals with the concepts that are needed to describe motion, without any reference to forces.
- **Dynamics** deals with the effect that forces have on motion.

Together, kinematics and dynamics form the branch of physics known as mechanics. In this chapter, we will deal with kinematics only and will concentrate on the simplest kin of motion: a body moving along a **straight line path**.

<table>
<tr><th>Distance</th><th>Displacement</th></tr>
<tr><td>Distance is defined as the actual (total) path between two points.
➢ Scalar quantity
➢ Always positive
➢ SI unit: meter (m)</td><td>Displacement is a vector drawn from the initial position to the final position. In other word, it is a straight line distance (shortest distance) from the initial position to the final position of an object.
➢ Vector quantity
➢ Can be positive, negative or zero
➢ SI unit: meter (m)</td></tr>
<tr><th colspan="2">Example</th></tr>
<tr><td colspan="2">



A car travelled from A to B, and then from B went back to C as shown in the diagram above.
Determine the distance and displacement travelled by the car.

Distance:

Displacement:</td></tr>
</table>

2.1 Linear Motion

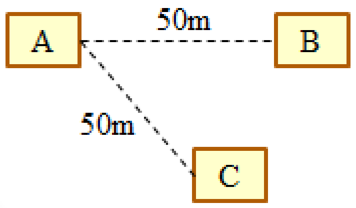
<table>
<tr><th>Average speed</th><th>Average velocity</th></tr>
<tr><td>
Average speed is defined as the distance travelled divided by the time required to cover the distance:

$$\text{Average speed} = \frac{\text{Distance}}{\text{Elapsed time}}$$

Explanation:
Term speed refer to how fast an object travels in a given time interval
➢ Scalar quantity
➢ Always positive
➢ SI unit: metre per second (m s^{-1})
</td><td>
Average velocity is defined as the displacement travelled divided by the elapsed time:

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Elapsed time}}$$

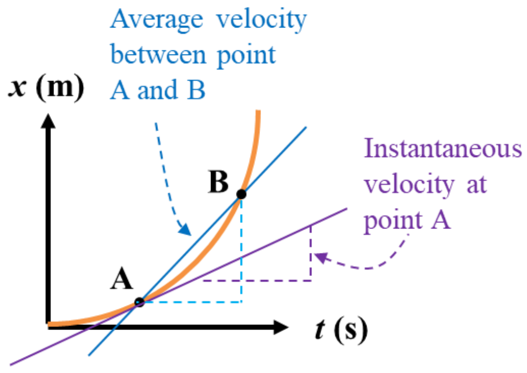
Explanation:
Term velocity tells us how fast object is moving and in which direction it is moving.
➢ Vector quantity
➢ Can be positive, negative or zero
➢ SI unit: metre per second (m s^{-1})
</td></tr>
<tr><th colspan="2">Example</th></tr>
<tr><td colspan="2">
Figure shows the distance AB and the distance AC is equal to 50m. A car P takes 10 s to move from A to B and another car Q takes 10s to moves from A to C.



The speed of both cars is the same because they have same magnitude. But the velocity of car P is not the same as the velocity of car Q because the direction of motion is not the same.
Speed of both car:
Velocity of car P:
Velocity of car Q:
</td></tr>
<tr><th>Instantaneous velocity</th><th>Uniform velocity</th></tr>
<tr><td>
The instantaneous velocity is the limit of the average velocity as the elapsed time approaches zero.

$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

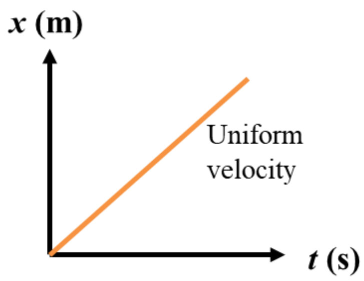
Explanation:
It indicates how fast the object moves and the direction of the motion at each instant of time



</td><td>
Moving with the same velocity at each instant of time.

$$v = \text{constant}$$

Explanation:
Magnitude is constant and direction remains unchanged.



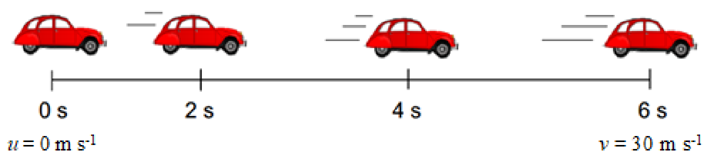
</td></tr>
</table>

Average acceleration	Instantaneous acceleration	Uniform acceleration
Average acceleration is define as the **change in velocity travelled divided by the elapsed time** $a_{av} = \dfrac{\text{Change in velocity}}{\text{Elapsed time}}$ $a_{av} = \dfrac{\Delta v}{\Delta t}$ $a_{av} = \dfrac{v_2 - v_1}{t_2 - t_1}$	The instantaneous acceleration is the limit of the average acceleration as the elapsed time approaches zero. $a = \lim\limits_{\Delta t \to 0} \dfrac{\Delta v}{\Delta t} = \dfrac{dv}{dt} = \dfrac{d^2 s}{dt^2}$ **Explanation:** It is the acceleration at a **specified position** or **at a particular time** along the path of motion.	Moving with the **same acceleration** at each instant of time $a = \text{constant}$ **Explanation:** Magnitude is constant and **direction remains unchanged**.

If the velocity of an object decreases from initial velocity, *u* to a lower final velocity, *v*, its acceleration is a **negative value** and it's called **deceleration/ retardation** (object is **slowing down –** direction of acceleration is **opposite** to the direction of motion or velocity).

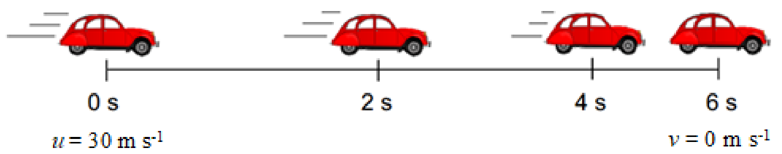
Summary

Acceleration, increasing velocity



$$a = \frac{30\,\text{m s}^{-1} - 0\,\text{m s}^{-1}}{6\,\text{s}} = 5\,\text{m s}^{-2}$$

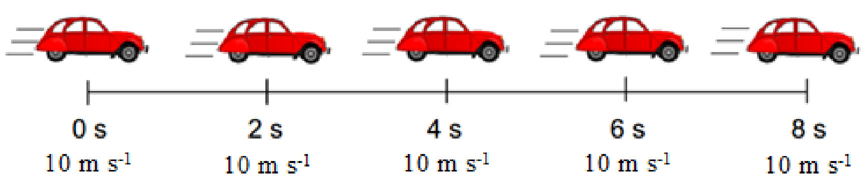
Deceleration, decreasing velocity



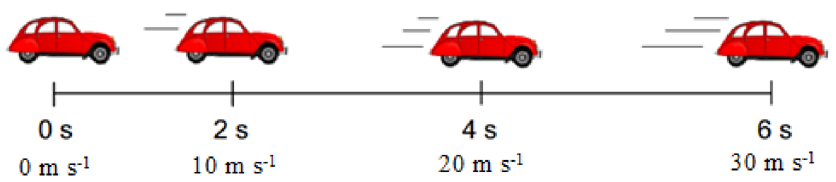
$$a = \frac{0\,\text{m s}^{-1} - 30\,\text{m s}^{-1}}{6\,\text{s}} = -5\,\text{m s}^{-2}$$

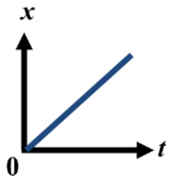
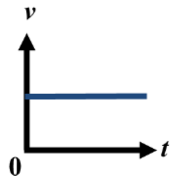
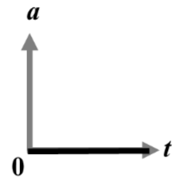
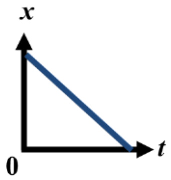
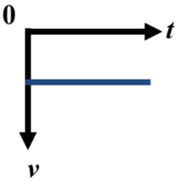
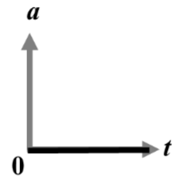
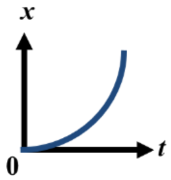
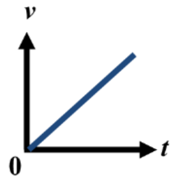
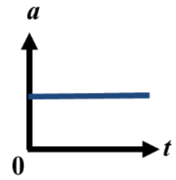
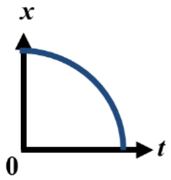
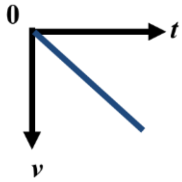
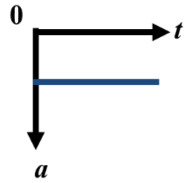
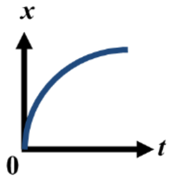
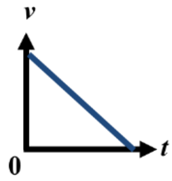
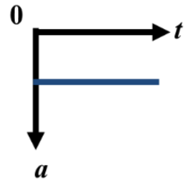
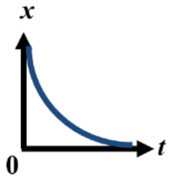
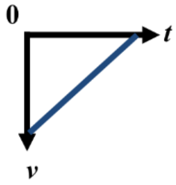
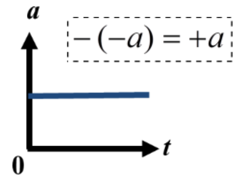
(**Negative** acceleration)

Zero acceleration, constant velocity



Constant Acceleration, velocity increase at constant rate



	Displacement-time (x - t)	**Velocity time (v – t)**	**Acceleration time (a - t)**
Remarks	i) Gradient of ***x–t*** graph = **velocity**	i) Gradient of ***v-t*** graph = **acceleration** ii) Area under ***v-t*** graph = **displacement**	i) Area under ***a-t*** graph = **velocity**
Uniform motion	 Gradient = constant	 Constant velocity	 No acceleration
	 Gradient = constant (Opposite direction)	 Constant velocity (Opposite direction)	 No acceleration
Non-uniform motion	 Gradient increases	 Velocity increases	 Object is under acceleration
	 Gradient increases (Opposite direction)	 Velocity increases (Opposite direction)	 Object is under acceleration (Opposite direction)
	 Gradient decreases	 Velocity decreases	 Object is under deceleration
	 Gradient decreases (Opposite direction)	 Velocity decreases (Opposite direction)	 Object is under deceleration (Opposite direction)

2.2 Uniform Accelerated Motion

The simplest kind of accelerated motion is straight line motion with constant acceleration. There are four equations of motions that can be used to solve any problem involving **straight-line motion** of an object with **constant acceleration**:

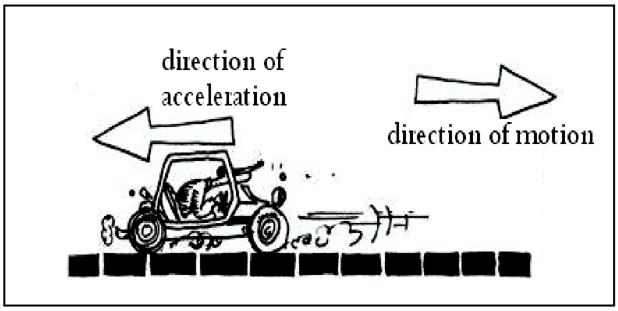
$$\boxed{v = u + at} \qquad \boxed{v^2 = u^2 + 2as}$$

$$\boxed{s = ut + \frac{1}{2}at^2} \qquad \boxed{s = \frac{1}{2}(u + v)t}$$

Sign convention:

Quantity	Horizontal Motion		Vertical Motion	
	+ve	**–ve**	**+ve**	**–ve**
Displacement, *s*	To the right	To the left	Above original position	Below original position
Velocity, *v*	To the right	To the left	Upward	Downward
Acceleration, *a*	Accelerate	Decelerate	Moves upward **with external force** such as engine (increasing velocity)	**Free fall** or slowing down (even if it moves upward)

Important:
- Uniform = constant = same
- Increasing velocity (speed up) = acceleration
- Decreasing velocity (slow down) = deceleration
- Zero velocity = the object is **stationary** (at rest)
- Negative velocity = the object moves in **opposite** direction
- Uniform velocity = zero acceleration
- Negative acceleration = deceleration (retardation) **OR** acceleration in the **opposite** direction.



Deceleration (negative acceleration)

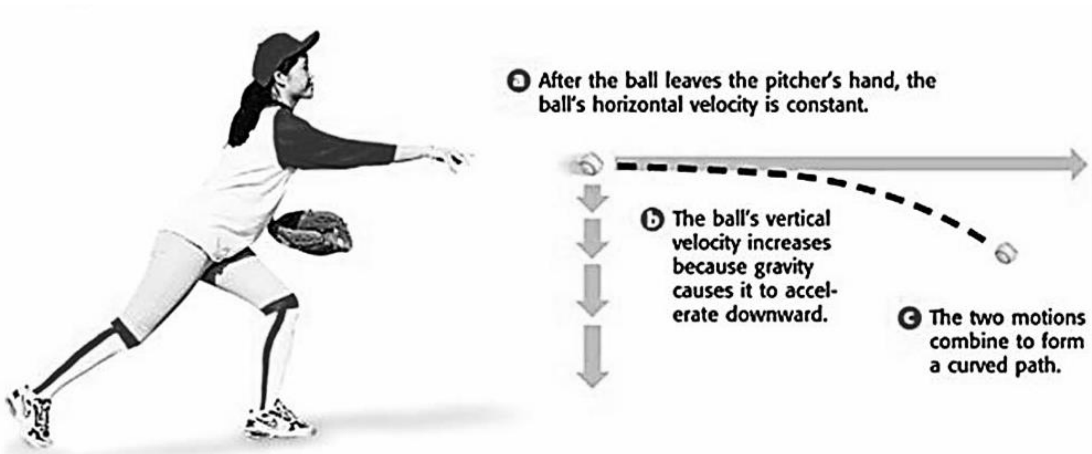
2.3 Projectile Motion

- The motion of an object that given an initial velocity and moves in space freely under the influence of gravity.
- A projectile motion consists of two components:
  - vertical component (**y-comp**.), motion under **constant acceleration**

$$\boxed{a_y = -g}$$

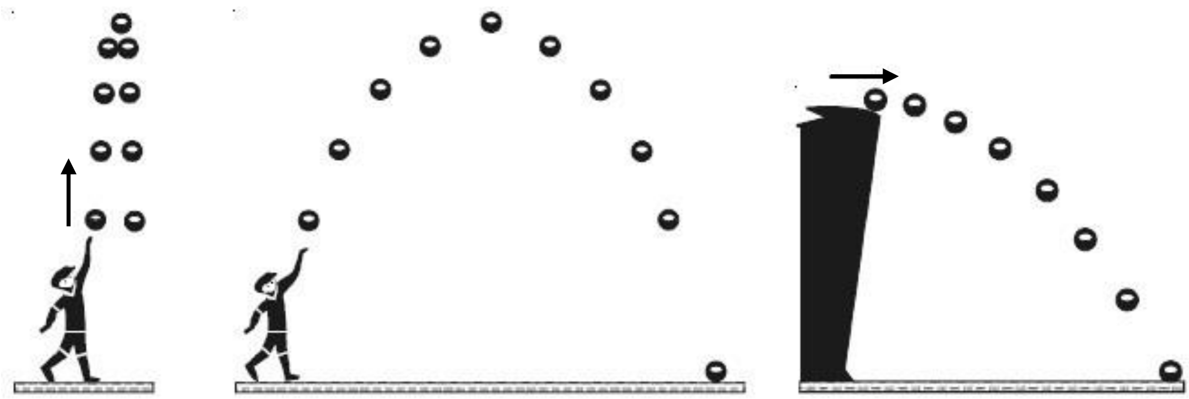
  - horizontal component (**x-comp**.), motion with **constant velocity** thus

$$\boxed{a_x = 0}$$



Projectile Motion

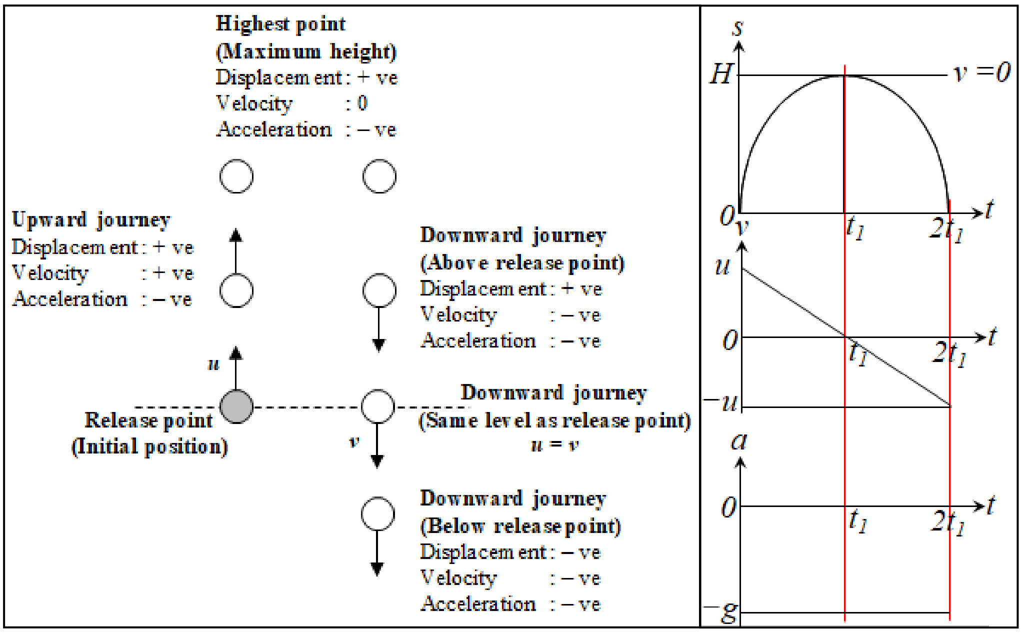
- **Assumptions** of projectile motion:
  - Free fall acceleration g is constant and is always directed downward.
  - Neglect air resistance.
- For calculation, we consider only its motion **after** it has been projected, and **before** it lands or is caught – that is, we analyse our projected object only when it is moving freely through the air under the action of gravity alone.
- Types of projectile motion:



$\theta = 90°$ (Free fall) | $0° < \theta < 90°$ | $\theta = 0°$ (Horizontal projectile motion)

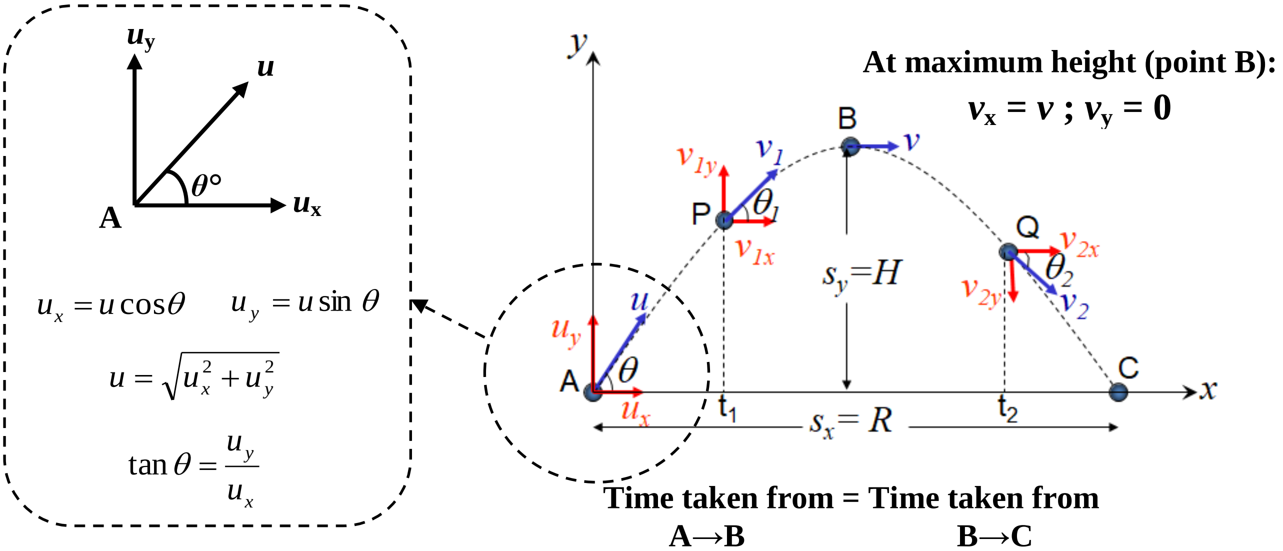
Special case: $\theta = 90°$ (Free fall motion)

- A free falling body is defined as the vertical motion of a body at constant acceleration, *g*, under gravitational field without air resistance.
- In the earth gravitational field, the constant acceleration
  - ✓ known as acceleration due to gravity
  - ✓ the **magnitude** (without any algebraic sign) of this acceleration is denoted as ***g*** and *is* approximately $\boldsymbol{g} = \mathbf{9.81\ m\ s^{-2}}$ near the earth's surface
  - ✓ the direction is towards the centre of the earth (**downward**)
- **Assumption:** Free falling objects **do not encounter air resistance**.
- Since the acceleration is constant in free-fall, the equations of kinematics can be used.
- Acceleration due to gravity is a vector, thus we use $\boldsymbol{a} = -\boldsymbol{g}$ in calculation because the direction is downward toward the center of the Earth.



Projectile motion at any angle ($0° < \theta < 90°$)

Consider an object thrown with a velocity, *u* at an angle $\theta°$ relative to the horizontal:

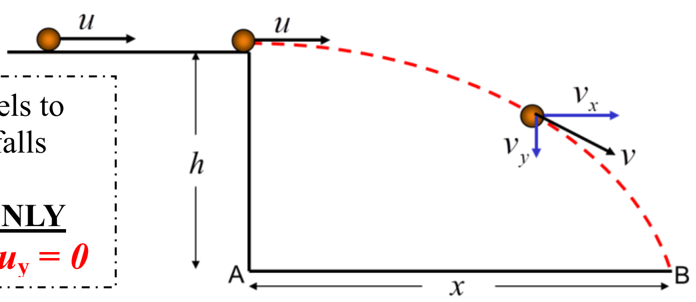


To solve problems on projectile motion, we treat the horizontal and vertical component separately, and we can apply the kinematics equations to the x and y components of the motions. Table shows the kinematic equations for projectile motion in x and y component.

Quantity	**Horizontal motion** **(x – component)**	**Vertical motion** **(y – component)**
Acceleration, a	$a_x = 0$	$a_y = -g$ $g = 9.81\,\mathrm{m\,s^{-2}}$ (constant)
Velocity, v	$v_x = u_x$ (constant)	**Free fall** $v_y = u_y - gt$ $v_y^2 = u_y^2 - 2gs$
Displacement, s	$s_x = u_x t - \cancelto{0}{\frac{1}{2}a_x t^2}$	$s_y = u_y t - \frac{1}{2}gt^2$

<u>Special case: $\theta = 0°$ (Horizontal projectile motion)</u>

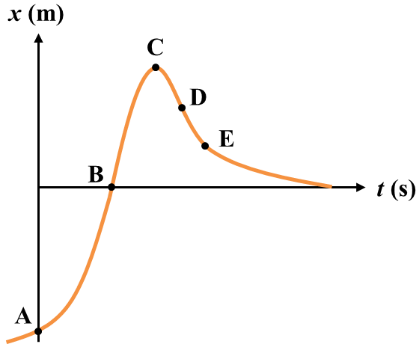
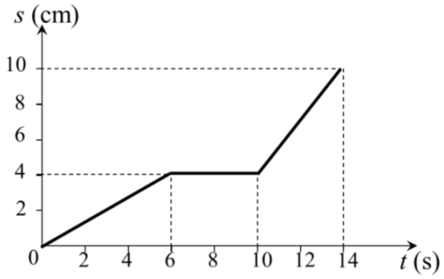
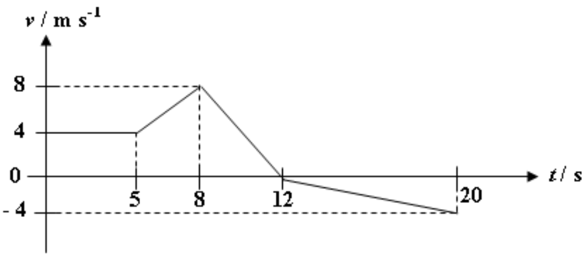
Projectile travels to the right as it falls downward.
Initially, we **<u>ONLY</u>** have $u_x = u, u_y = 0$



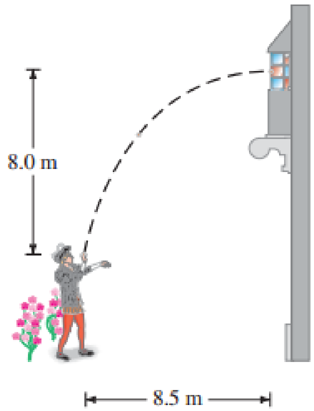
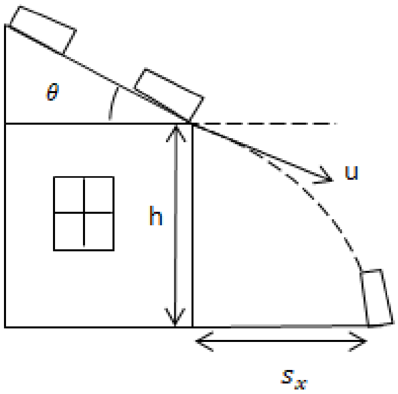
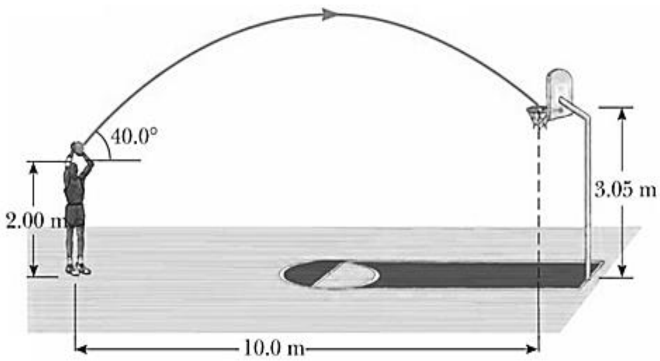
Quantity	**Horizontal motion** **(x – component)**	**Vertical motion** **(y – component)**
Acceleration, a	$a_x = 0$	$a_y = -g$ $g = 9.81\,\mathrm{m\,s^{-2}}$ (constant)
Velocity, v	$v_x = \boxed{u_x = u}$ (constant)	**Free fall** $v_y = \cancelto{0}{u_y} - gt$ $v_y^2 = \cancelto{0}{u_y^2} - 2gs$
Displacement, s	$s_x = u_x t - \cancelto{0}{\frac{1}{2}a_x t^2}$	$s_y = \cancelto{0}{u_y t} - \frac{1}{2}gt^2$

Exercise

Revision	
1.	A honeybee leaves the hive and travels a total distance of 2 km before returning to the hive. a. What is the distance travelled by the bee? b. What is the displacement travelled by the bee?
2.	One afternoon, a couple walks three-fourths of the way around a circular lake, the radius of which is 1.50 km. They start at the west side of the lake and head due south to begin with. a. Calculate the distance travelled by the couple. a. Determine the magnitude and direction (relative to due east) of the couple's displacement.

Linear Motion	
1.	 Explain qualitatively the motion of the objects at point **A**, **B**, **C**, **D**, and **E** labelled on the *x*-*t* graph.
2.	A toy train moves slowly along a straight track according to the displacement, *s* against time, *t* graph in figure below.  a. Determine the average velocity for the whole journey. b. Calculate the instantaneous velocity at t = 12 s.
3.	Figure below shows a velocity-time graph of a toy car.  a. Calculate the average velocity and average speed of the entire motion. b. Sketch the shape of the corresponding acceleration-time graph.

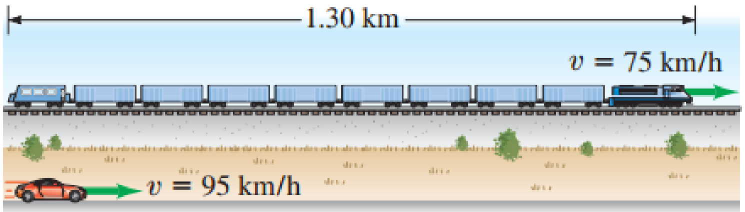
Uniform Accelerated Motion	
1.	A plane lands on a runway at velocity 50 m s^{-1} and decelerates at constant rate. The plane travels 1.0 km before **stops**. Calculate a. the deceleration of the plane. b. the time taken for the plane to stop.
2.	A bus travelling steadily at 30 m s^{-1} along a straight road passes a **stationary** car which, 5 s later, begins to move with a uniform acceleration of 2 m s^{-2} in the same direction as the bus. Determine a. the time taken for the car to acquire the same velocity as the bus. b. the distance travelled by the car when it is level with the bus.
3.	The speed limit in a school zone is 40 km h^{-1}. A driver traveling at this speed sees a child run onto the road 13 m ahead of his car. He applies the brakes and the car decelerates at a uniform rate of 8.0 m s^{-2}. If the driver's **reaction time** is 0.25 s, will the car stop before hitting the child?
4.	A book is **dropped** 150 m from the ground. Determine a. the time taken for the book reaches the ground. b. the velocity of the book when it reaches the ground.
5.	A ball is **thrown** directly downward, with an initial speed of 8.00 m s^{-1}, from a height of 30.0 m. Calculate a. the time taken for the ball to strike the ground. b. the ball's speed when it reaches the ground.
6.	A boy throws a stone straight upward with an initial speed of 15 m s^{-1}. What **maximum height** will the stone reach before falling back down?
7.	A rocket is launched upwards from ground with initial velocity of 35 m s^{-1}. The magnitude of its **acceleration is 5.0 m s^{-2}**. Determine the speed of the rocket at height 20 km.
8.	Two soccer players start from rest, 48 m apart. They run directly toward each other, both players accelerating. The first player's acceleration has a magnitude of 0.50 m s^{-2}. The second player's acceleration has a magnitude of 0.30 m s^{-2}. a. How much time passes before the players collide? b. At the instant they collide, how far has the first player run?
9.	Two stones are thrown simultaneously, one straight upward from the base of a cliff and the other straight downward from the top of the cliff. The height of the cliff is 6.00 m. The stones are thrown with the same speed of 9.00 m s^{-1}. Find the location (above the base of the cliff) of the point where the stones cross paths.
10.	A van driver is moving at a uniform velocity of 20 m s^{-1} heading toward an overhead bridge of the height of 10 m. When the van is 40 m from the overhead bridge, the van driver sees the signboard from the overhead bridge starts to fall. a. Calculate the time taken for the signboard to hit the ground. b. Assume that the length of the van is 6.0 m and ignore the height of van, determine whether the signboard will hit the van or not for the following cases: i. if the driver makes a decision to applied brake with deceleration of 5.3 m s^{-2}. ii. if the driver makes a decision to accelerate his van at a rate of 16.4 m s^{-2}.

	Projectile Motion
1.	A ball is thrown **vertically upward**, which is the positive direction. A little later it returns to its point of release. The ball is in the air for a total time of 8.0 s. What is its initial velocity? Neglect air resistance.
2.	A **horizontal rifle** is fired at a bull's-eye. The muzzle speed of the bullet is 670 m s^{-1}. The gun is pointed directly at the center of the bull's-eye, but the bullet strikes the target 0.025 m below the center. What is the horizontal distance between the end of the rifle and the bull's-eye?
3.	An object is projected with an initial velocity of 20.0 m s^{-1} and **at an angle of 30°** to the horizontal. Assuming that the acceleration due to gravity is 9.81 m s^{-2}, determine the a. **maximum height** reached b. time of flight c. **range** of the object
4.	Romeo is throwing pebbles gently up to Juliet's window, and he wants the pebbles to **hit the window with only a horizontal component of velocity**. He is standing at the edge of a rose garden 8.0 m below her window and 8.5 m from the base of the wall (Figure). How fast are the pebbles going when they hit her window? 
5.	Santa parked his sleigh 5.0 m from the edge of a 20° roof. Unfortunately, the parking brake failed. Santa's sleigh slid down the roof and landed on the ground 6.0 m from the wall of the house. Given that the velocity of the sleigh after sliding 5.0 m is 5.11 ms^{-1}. How high was the wall of the house? 
6.	A basketball player who is 2.00 m tall is standing on the floor 10.0 m from the basket, as in figure. If he shoots the ball at a 40.0° angle above the horizontal, at what initial speed must he throw so that it goes through the hoop without striking the backboard? The basket height is 3.05 m. 

7. A ball is shot from the top of a building with an initial velocity of 18 m s^{-1} at an angle θ = 42° above the horizontal.
   a. What are the horizontal and vertical components of the initial velocity?
   b. If a nearby building is the same height and 55 m away, how far below the top of the building will the ball strike the nearby building?

HOTS Questions

1. An automobile traveling 95 km h^{-1} overtakes a 1.30 km long train traveling in the same direction on a track parallel to the road. If the train's speed is 75 km h^{-1}.
   a. How long does it take the car to pass it, and how far will the car have travelled in this time?
   b. What are the results if the car and train are traveling in opposite directions?



2. An unmarked police car travelling a constant 95 km h^{-1} is passed by a speeder travelling 140 km h^{-1}. Precisely after 1.00 s after the speeder passes, the policemen steps on the accelerator, if the police car's acceleration is 2.00 m s^{-2}, how much time passes before the police car overtakes the speeder (assumed moving at constant speed)?

3. Mary and Sally are in a foot race. When Mary is 22 m from the finish line, she has a speed of 4.0 m s^{-1} and is 5.0 m behind Sally, who has a speed of 5.0 m s^{-1}. Sally thinks she has an easy win and so, during the remaining portion of the race, decelerates at a constant rate of 0.40 m s^{-2} to the finish line. What constant acceleration does Mary now need during the remaining portion of the race, if she wishes to cross the finish line side-by-side with Sally?

4. While standing on a bridge 15.0 m above the ground, you drop a stone from rest. When the stone has fallen 3.20 m, you throw a second stone straight down. What initial velocity must you give the second stone if they are both to reach the ground at the same instant? Take the downward direction to be the negative direction.

5. Two cannons are mounted as shown in the drawing and rigged to fire simultaneously. They are used in a circus act in which two clowns serve as human cannonballs. The clowns are fired toward each other and collide at a height of 1.00 m above the muzzles of the cannons. Clown A is launched at a 75.0° angle, with a speed of 9.00 m s^{1}. The horizontal separation between the clowns as they leave the cannons is 6.00 m. Find the launch speed v_{0B} and the launch angle θ_B (>45.0°) for clown B.