# Chapter 6  Circular Motion

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<td><strong>6.1 Uniform Circular Motion</strong></td>
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<td>a) Describe uniform circular motion. (C1, C2)</td>
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<td>b) Convert units between degrees, radian, and revolution or rotation. (C3, C4)</td>
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<td><strong>6.2 Centripetal Force</strong></td>
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<td>a) Define centripetal acceleration. (C1, C2)</td>
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<td>b) Solve problems related to centripetal force for uniform circular motion cases: horizontal circular motion, vertical circular motion and conical pendulum. (C3, C4)</td>
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6.1 **Uniform Circular Motion**

- Uniform circular motion is the motion of an object traveling at a constant (uniform) speed on a circular path.
- The **magnitude** of the velocity remains constant in this case, but the **direction** of the velocity continuously changes as the object moves around the circle.

- Circular motion is often described in terms of the **frequency** \( f \), the **number of revolutions per second** (rps).
- The **period** \( T \) of an object revolving in a circle is the time required for one complete revolution.
- Period and frequency are related by:

\[
T = \frac{1}{f}
\]

- There is a relationship between period and speed, since speed \( v \) is the distance travelled (here, the circumference of the circle = \( 2\pi r \)) divided by the time \( T \):

\[
v = \frac{2\pi r}{T}
\]

**Relationship between tangential variables and angular variables**

<table>
<thead>
<tr>
<th>Distance, ( s )</th>
<th>Angular displacement, ( \theta )</th>
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<tr>
<td>The <strong>actual (total) path</strong> between two points.</td>
<td>When a rigid body rotates about a fixed axis, the angular displacement is the angle ( \Delta \theta ) swept out by a line passing through any point on the body and intersecting the axis of rotation perpendicularly.</td>
</tr>
<tr>
<td>Unit: ( m )</td>
<td>Unit: ( \text{rad} )</td>
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\[
s = r\theta
\]
**Tangential speed, \( v \)**

- The distance travelled divided by the elapsed time.
  \[ v = \frac{s}{t} \]
- Unit: \( \text{m s}^{-1} \)
- The direction of tangential speed, \( v \) is always tangential to the circle.

**Angular velocity, \( \omega \)**

- The angular displacement divided by the elapsed time.
  \[ \omega = \frac{\theta}{t} \]
- The magnitude of the angular velocity is known as the angular frequency.
- Unit: \( \text{rad s}^{-1} \)
- The direction of the angular velocity, \( \omega \) is depending on the rotation of the object (clockwise or anticlockwise).

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<tr>
<th>Tangential speed, ( v )</th>
<th>Angular velocity, ( \omega )</th>
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<td>The distance travelled divided by the elapsed time.</td>
<td>The angular displacement divided by the elapsed time.</td>
</tr>
<tr>
<td>[ v = \frac{s}{t} ]</td>
<td>[ \omega = \frac{\theta}{t} ]</td>
</tr>
<tr>
<td>Unit: ( \text{m s}^{-1} )</td>
<td>Unit: ( \text{rad s}^{-1} )</td>
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<td>The direction of tangential speed, ( v ) is always tangential to the circle.</td>
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The right-hand rule for determining the direction of the angular velocity vector.
Thumb → direction of \( \omega \)
Fingers → direction of rotation

\[ v = r \omega \]

---

**Unit**

- 1 revolution (rev) = 1 rotation = 1 complete cycle = \( 2\pi \) rad = 360°
  Time taken to complete 1 revolution is called **period, \( T \)**.

- The circumference of a complete cycle = \( 2\pi r \)

- Relationship between **radian** and **degree**:
  \( \pi \) rad = 180° = half revolution
  \( 2\pi \) rad = 360° = 1 revolution

- 1 revolution per second (rps) = \[ \frac{1 \text{ revolution}}{1 \text{ second}} = \frac{2\pi}{1} = 2\pi \text{ rad s}^{-1} \]

- 1 revolution per minute (rpm) = \[ \frac{1 \text{ revolution}}{60 \text{ second}} = \frac{2\pi}{60} = 0.1 \text{ rad s}^{-1} \]


### 6.2 Centripetal Force

**Additional Knowledge: Derivation**

- Centripetal acceleration refers to the acceleration of an object moving in a circle of radius $r$ at constant speed $v$ has an acceleration whose **direction is toward the centre of the circle** and whose **magnitude** is

  $$ a_C = \frac{v^2}{r} $$

- The centripetal force is the name given to the net force required to keep an object of mass $m$, moving at a speed $v$, on a circular path of radius $r$, and it has a **magnitude** of

  $$ F_C = ma_C = \frac{mv^2}{r} $$

- The centripetal force always **points toward the centre of the circle** and continually changes direction as the object moves.

- To find the instantaneous acceleration, $\Delta t \approx 0$, thus $\Delta \theta \approx 0$.
- When $\Delta \theta \approx 0$, $\Delta l = s$.
- Therefore, the ratio of

  $$ \frac{v}{\Delta v} = \frac{r}{\Delta l} = \frac{r}{v \Delta t} $$

- Centripetal acceleration

  $$ a_C = \frac{\Delta v}{\Delta t} = \frac{v^2}{r} $$
• If the centripetal force suddenly stops to act on a body in the circular motion, the body flies off in a straight line with the constant tangential (linear) speed.

• Centripetal force is just the name we give to the net force acting on the object in a direction towards the centre of the circle.
• Centripetal force is NOT the new force acting on the object, therefore DO NOT draw the centripetal force in the free body diagram.

Uniform Circular Motion Cases

*Horizontal Circular Motion*

**Example 1**: Object revolving in a horizontal circle with steady speed.

\[
F_C = \frac{mv^2}{r} \\
T = \frac{mv^2}{r}
\]

**Example 2**: Motion of car round a flat curve.

\[
F_C = \frac{mv^2}{r} \\
f = \frac{mv^2}{r}
\]
Vertical Circular Motion

Example 1: A ball is attached to an inelastic string and moves in a vertical circle.

At the top
\[ F_C = \frac{mv^2}{r} \]
\[ T + mg = \frac{mv^2}{r} \]

THINK
How to determine the minimum speed needed to keep the ball swinging in the vertical circle?

Example 2: Riding a Ferris wheel at constant speed.

At the top
\[ F_C = \frac{mv^2}{r} \]
\[ mg - N = \frac{mv^2}{r} \]

THINK
- Which position would give smaller reading on the weighing scale?
- How to determine the maximum speed at which the Ferris wheel can be rotated before the object/human flew off the Ferris wheel?
Conical Pendulum

\[ F_C = \frac{mv^2}{r} \]
\[ T \cos \theta = \frac{mv^2}{r} \]

\[ \tan \theta = \frac{\sin \theta}{\cos \theta} \]

\[ F_C = 0 \] along the \( y \) – component because circular motion only occurred along the \( x \) – component.

Try it yourself!

\[ x – component \]
\[ y – component \]
# Exercise

## Uniform Circular Motion

1. An object undergoes circular motion with uniform angular speed 300 rpm. Determine
   a) the period, $T$
   b) the frequency, $f$

2. Determine the angular velocity of the hour–hand and minute-hand of a clock.

3. A particle moving in a circle takes 0.25 s to complete one revolution. How many complete revolutions per minute (rpm) does the particle perform?

4. The wheel of a car has a radius of $r = 0.29$ m and is being rotated at 830 revolutions per minute (rpm) on a tire-balancing machine. Determine the tangential velocity at which the outer edge of the wheel is moving.

5. Two wheels of a machine are connected by a transmission belt. The radius of the first wheel $r_1 = 0.60$ m, the radius of the second wheel $r_2 = 0.13$ m. The frequency of the bigger wheel equals 4.5 Hz. What is the frequency of the smaller wheel?

## Centripetal Force

1. A car travels at a constant speed around a circular track whose radius is 2.6 km. The car goes once around the track in 360 s. What is the magnitude of the centripetal acceleration of the car?

2. A child sitting 1.20 m from the centre of a merry-go-round moves with a speed of 1.10 m s$^{-1}$. Calculate
   a) the centripetal acceleration of the child
   b) the net horizontal force exerted on the child ($m_{\text{child}} = 22.5$ kg)

3. A 0.55 kg ball, attached to the end of a horizontal cord, is revolved in a circle of radius 1.3 m on a frictionless horizontal surface. If the cord will break when the tension in it exceeds 75 N, what is the maximum speed the ball can have?

4. A coin placed 30.0 cm from the center of a rotating, horizontal turntable slips when its speed is 50.0 cm s$^{-1}$. What is the coefficient of static friction between coin and turntable?

5. A bucket of mass 2.00 kg is whirled in a vertical circle of radius 1.20 m. At the lowest point of its motion the tension in the rope supporting the bucket is 25.0 N.
   a) Find the speed of the bucket.
   b) How fast must the bucket move at the top of the circle so that the rope does not go slack?

6. Suppose a 1800 kg car passes over a hump in a roadway that follows the arc of a circle of radius 20.4 m as shown in Figure.
   a) If the car travels at 30 km h$^{-1}$, what force does the road exert on the car as the car passes the highest point of the hump?
   b) What is the maximum speed the car can have without losing contact with the road as it passes this highest point?
7. A “swing” ride at a carnival consists of chairs that are swung in a circle by 15.0 m cables attached to a vertical rotating pole, as the drawing shows. Suppose the total mass of a chair and its occupant is 179 kg.
   a) Determine the tension in the cable attached to the chair.
   b) Find the speed of the chair.

8. The object of mass $m = 4.0$ kg in Figure is attached to a vertical rod by two strings of length $l = 2.0$ m. The strings are attached to the rod at points a distance $d = 3.0$ m apart. The object rotates in a horizontal circle at a constant speed of $v = 3.0$ m s$^{-1}$, and the strings remain taut. The rod rotates along with the object so that the strings do not wrap onto the rod. Determine the tension in each string.

**HOTS Questions**

1. Ball A is attached to one end of a string, while an identical ball B is attached to the centre of the string, as Figure below illustrates. Each ball has a mass of $m = 0.50$ kg and the length of each half of the string is $L = 0.40$ m. This arrangement is held by the empty end and is whirled around in a horizontal circle at a constant rate, so each ball is in uniform circular motion. Ball A travels at a constant speed of $v_A = 5.0$ m s$^{-1}$ and ball B travels at a constant speed of $v_B = 2.5$ m s$^{-1}$. Find the tension in each half of the string.

2. An object of mass $m_1 = 4.0$ kg is tied to an object of mass $m_2 = 3.0$ kg with String 1 of length $l = 0.5$ m. The combination is swung in a vertical circular path on a second string, String 2 of length $l = 0.5$ m. During the motion, the two strings are collinear at all times as shown in Figure. At the top of its motion, $m_2$ is travelling at $v = 4.0$ m s$^{-1}$.
   a) What is the tension in String 1 at this instant?
   b) What is the tension in String 2 at this instant?
   c) Which string will break first if the combination is rotated faster and faster?